STT 873 HW2

This HW is due on Sep 25th.

Question 1: Prove the inner product $< \cdot, \cdot >$ defined in the RKHS is a well defined inner product by verifying the following properties

(1) Symmetric: $< f, g >=< g, f >$.

(2) Linear: $< cf + dg, h >= c < f, h > +d < g, h >$

(3) $< f, f >= 0$ iff $f = 0$

Question 2 (email the R code directly to zhang318@stt.msu.edu and xyy@egr.msu.edu):

(1) Write a R function to solve the following RKHS regression problem.

$$\hat{f} = \text{argmin}_{f \in RKHS} \sum_{i=1}^{n} (y_i - f(x_i))^2 + \lambda \|f\|_K^2,$$

where $\| \cdot \|_K$ is the norm defined in the RKHS space with Gaussian kernel ($K(x, y') = \exp(-\|x - y'\|^2/0.25)$), Laplacian kernel $K(x, y') = \exp(-\|x - y'\|)$,or Polynomial kernel ($K(x, y') = (< x, y' > +1)^2$). (You can also compare your results with those using the R pacakge 'KERE').

(2) Download data from 'goo.gl/pkBTsy', and test your function with $\lambda = 0.5$ and 0.01 for Gaussian and Laplacian kernels and $\lambda = 0.5$ and 0.2 for Polynomial kernel. Plot X vs $\hat{Y}$ on top of the original data points.